



How simple can nonlinear finite element modelling be for structural concrete?

¿Cuál debería ser el nivel de sencillez ideal para un análisis no lineal de elementos en hormigón estructural?

G. Argirova (*), M. Fernández Ruiz (*), A. Muttoni (*)

ABSTRACT

This paper discusses on the required level of simplicity for suitable modelling of structural concrete. Traditional equilibrium-based approaches (as strut-and-tie models) are too coarse in some cases, as they account for the cracking state of concrete in a sometimes excessively simplified manner. The alternative of complex nonlinear numerical modelling is also not always satisfactory for design as the number of parameters required, their definition and the sensitivity of the structural response to them is complex and requires a high level of experience. Contrary to these approaches, this paper introduces the elastic plastic stress field method. This method is grounded on the theory of plasticity but allows considering deformation compatibility. The results are consistent both in terms of the strength and deformation field of the member. It also has the advantage of requiring only two physical material properties (modulus of elasticity and plastic strength) which can be easily determined by designers.

Keywords: Structural concrete analysis; non-linear finite element modelling; shear critical beams; support zone; dapped-end beams.

RESUMEN

Este artículo discute sobre el nivel de sencillez ideal para un análisis no lineal de elementos de hormigón estructural. Los métodos de cálculo basados únicamente en condiciones de equilibrio (como los modelos de bielas-y-tirantes) no son siempre adecuados ya que el estado de fisuración del hormigón se considera a veces de una manera excesivamente simplificada. Los análisis no lineales complejos tampoco son siempre adecuados, ya que el número de parámetros requeridos, su definición y la sensibilidad de la respuesta del elemento a sus variaciones requieren una gran experiencia. Como alternativa, se presenta el método de los campos de tensiones elasto-plásticos. Este método se basa en la teoría de la plasticidad pero incorporando condiciones de compatibilidad. Los resultados son coherentes en términos de resistencia y de deformaciones. Además, sólo necesita la definición de dos parámetros mecánicos (módulo de elasticidad y resistencia plástica) que pueden ser fácilmente determinados por el proyectista.

Palabras clave: Hormigón estructural; modelización no lineal; vigas fallando por cortante; zona de apoyo; apoyo a media madera.

(*) ENAC - Ecole Polytechnique Fédérale de Lausanne (Switzerland).

Persona de contacto/Corresponding author: miguel.fernandezruiz@epfl.ch

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1. INTRODUCTION

Critical details of a structure are usually governing for the strength of the member. Such critical details of structures can be found most often where geometrical discontinuities occur or near the region where concentrated loads are applied. The prediction of the load-bearing capacity of such details is very important as well as of the associated failure mode, which may be potentially brittle (associated to localization of strains) or ductile (associated to smeared strains and reinforcement yielding).

The assessment of critical details of existing RC structures with sufficient amount of transverse reinforcement is generally performed for tender or executive design on the basis of equilibrium-based models such as strut-and-tie models or rigid plastic stress-fields. These methods have proven over time to give a clear and accurate insight of force transfer and ultimate capacity of structural concrete elements. However, for complicated or unusual details they require a certain level of experience in order to suitably account for the various load-carrying actions. In addition, these methods do not consider compatibility conditions (they refer to rigid-plastic material laws) and therefore cannot accurately account for some phenomena (such as strength reduction due to transverse cracking) or predict the deformation capacity of a structure. Some of these shortcomings, particularly with reference to the estimate of the crushing strength of a compression field accounting for its cracking state, have been overcome by some approaches as the Modified Compression Field Theory or the Rotating-Angle Softened Truss Model. These latter approaches demonstrate that accounting for the actual strain field of concrete (by means of compatibility conditions) is necessary to lead to consistent estimates of the strength and deformation capacity of concrete members.

Other than the equilibrium-based approaches (traditionally associated to hand-made calculations), design can be performed using nonlinear numerical modelling. This technique has been significantly developed in the last decades, leading to advanced computer programs based on different theories and accounting for a large number of effects in concrete and steel. The use of advanced numerical tools for design and assessment of these regions is becoming increasingly popular. This has been particularly encouraged in new Model Code 2010, whose design philosophy refers to the Levels-of-Approximation approach. This allows the use of simple design models for preliminary or tender design (low-order Levels-of-

Approximation) but allows refining it for advanced analyses and assessments (higher-order Levels-of-Approximation). The use of numerical tools consistent with the design procedures used for simpler analyses becomes thus encouraged for designers. A question arises then on how should a suitable nonlinear analysis be performed and how simple should it be. Two strategies are possible, the former consists on the use of the most complete and refined available constitutive laws. The latter consists on the use of the simpler and most understandable constitutive laws that, nevertheless, lead to accurate and consistent results.

With respect to the former strategy, these tools generally give a good prediction of deformation and strength capacities as well as of the potential load-carrying actions. However, they often require defining a large number of parameters in order to provide an accurate solution. Defining these parameters is sometimes cumbersome and the response of the program can potentially be rather sensitive to their choice. Furthermore, such analyses are associated to time-consuming analyses for pre- and post-processing of the data and may not be strictly consistent to the design methods used for simpler (low-order Levels-of-Approximation) analyses and their safety format.

An alternative to complex numerical modelling of structural concrete members with sufficient transverse reinforcement was recently developed at Ecole Polytechnique Fédérale de Lausanne. This technique is based on the analysis of elastic-plastic stress fields (EPSF). It shares the ground of equilibrium-based models (being thus consistent to low-order Levels-of-Approximation and their safety format) but allows accounting for compatibility conditions thus allowing to suitably represent the strain state of a concrete member and to refine some of the assumptions of strut-and-tie or rigid-plastic stress fields. For the analysis of the strength only two material parameters are required (modulus of elasticity and plastic strength) as well a strength reduction law accounting for transverse cracking. The method has proven to give excellent estimates of the strength and failures modes for a wide variety of structural concrete members.

The EPSF method has already been verified for several critical details such as deep beams (1), beams with openings (2), prestressed girders (3) (4), corners of frames (5) and light-weight concrete structures (6). This paper highlights the simplicity of the method and the good level of accuracy for the analysis of support zones of beams, dapped-end and shear-critical beams (Figure 1).

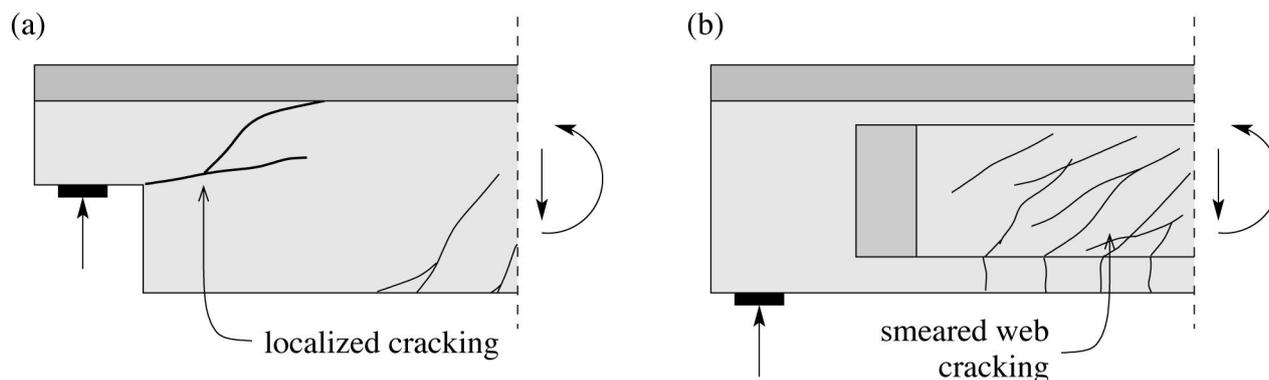


Figure 1. Critical details: (a) dapped end beams; and (b) support zones.

2. THE FUNDAMENTALS OF THE EPSF METHOD

2.1. Material models

The EPSF method assumes a reinforced concrete member to be provided with a sufficient amount of transverse reinforcement (1) and assumes the materials to behave in an elastic-plastic manner (1) (with potentially strain hardening). To do so, only the plastic strength and the modulus of elasticity of the materials are required. For the concrete, additionally, the following assumptions are performed:

- Mohr-Coulomb yield surface for plane stress with tension cut-off (consistent with rigid-plastic stress field assumptions with no tensile strength, equivalent for plane stress problems to a Rankine failure surface with no tensile strength).
- Consideration of brittleness of high strength concrete through a strength reduction parameter (η_{fc}):

$$f_{cp} = f_c \cdot \eta_{fc} \tag{1}$$

This parameter can be evaluated according to Muttoni (7) as:

$$\eta_{fc} = \left(\frac{30}{f_{ck}} \right)^{1/3} \leq 1 \tag{2}$$

- Consideration of concrete strength reduction with transverse cracking (k_c , Vecchio-Collins effect (8)):

$$k_c = \frac{1}{0,8 + 170 \cdot \epsilon_1} \tag{3}$$

For the steel a bilinear constitutive law, symmetrical in compression and tension, is adopted with the possibility to account for strain-hardening as shown in Figure 2c. Between reinforcement and concrete, perfect bond is assumed (non-linear springs accounting for actual bond behaviour being an alternative) with no tension-stiffening.

Since no tensile strength is considered, the stiffness of the member is usually underestimated at low levels of load. However, as load increases, the actual stiffness of the member is controlled by the available reinforcement and good predictions of the stiffness and strain state are obtained (1). Following this assumption, the EPSF method is shown to provide

accurate estimates of the strength of members with sufficient transverse reinforcement, and to lead to safe estimates of the strength for members without transverse reinforcement (5).

It can be noted that no fracture mechanics concepts are in-built in the stress field (tensile strength of concrete is set to zero) and that the design principles are consistent to those of rigid-plastic stress fields or strut-and-tie models (used for preliminary or tender designs). This allows a significant level of consistency in the results (and safety format) obtained from first stages of design to the most refined ones. With respect to its accuracy, since the EPSF incorporates compatibility conditions, at failure, the resulting stress field is compatible with a licit failure mechanism. This provides an exact solution according to the theory of plasticity (lower and upper bound simultaneously), leading to the best potential stress field that can be selected.

2.2. Numerical implementation

The EPSF method can be efficiently implemented through the Finite Element Method (FEM) (1). Concrete is modelled by means of constant strain triangle elements for which over a given displacement field the principal strains and their directions can be found. An important assumption of the method is that the direction of the principal strains and stresses are parallel (fully-rotational model). Therefore, using the bilinear material model, described in the previous section, the principal stresses can be obtained and the nodal forces in the principal directions are determined from (1):

$$F_{j,i} = \sigma_i \cdot \left(\frac{l_j}{2} \cos(\beta_j) - \frac{l_{j+1}}{2} \cos(\beta_{j+1}) \right) \tag{4}$$

for $i \in (1,2)$ and $j \in (1,3)$

where l_j is the side of the element and the angle β_j is defined in Figure 3.

The reinforcement is modelled by a two node link element with no transversal stiffness (thus neglecting dowel action). Using the bilinear material model, described in the previous section, and provided a displacement field at the nodes, the axial forces can be obtained.

By introducing a small incremental displacement at each node and direction the stiffness coefficients can be found the stiffness matrix formed. The solution is found through a full

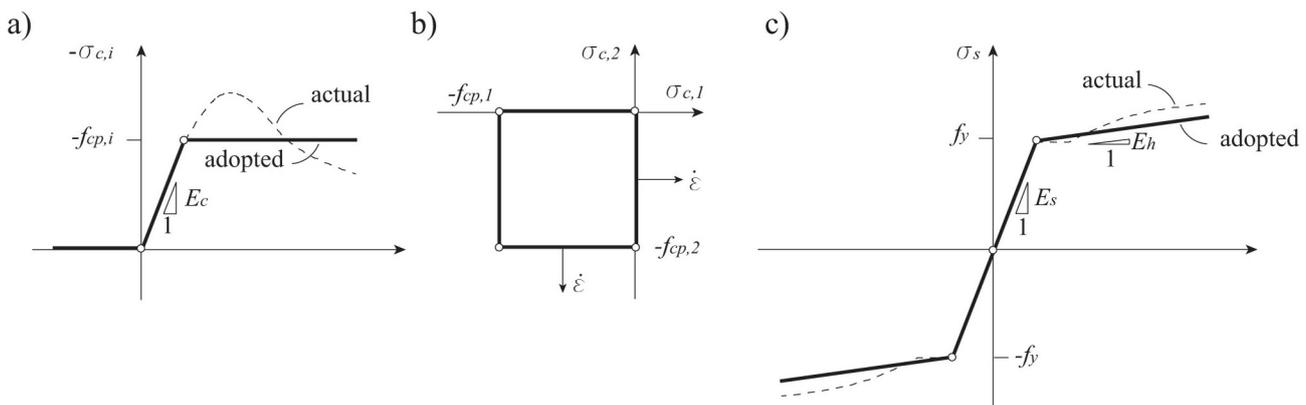


Figure 2. (a) constitutive model for the concrete; (b) Mohr-Coulomb yield surface; and (c) constitutive model for the steel.

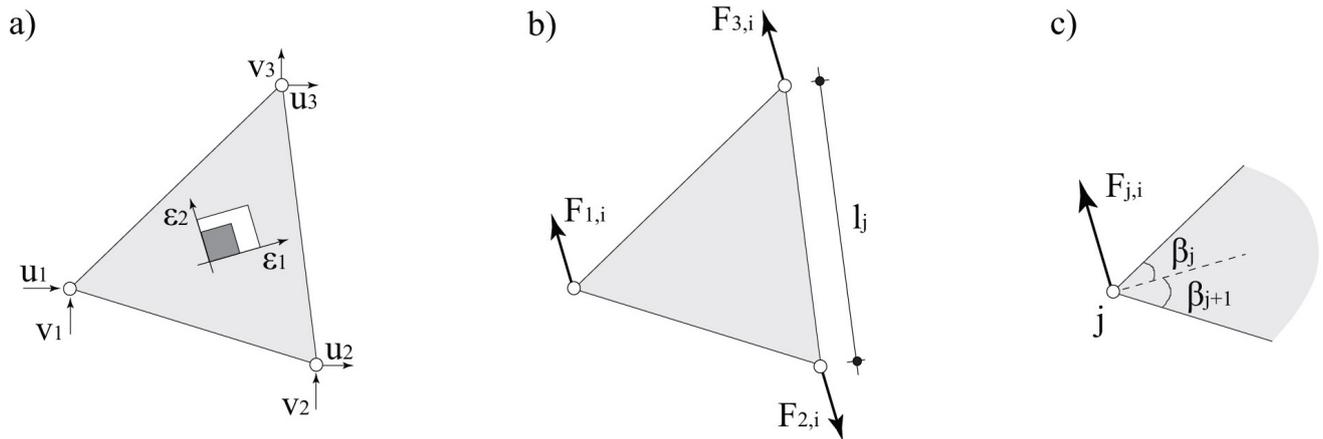


Figure 3. Constant strain triangle.

Newton-Raphson solver, providing quadratic convergence to the solution. The Elastic Plastic Stress Field (EPSF) has been implemented into an open Java computer program that can be run standalone or as an applet of a web-page. The access for students at EPFL and practicing engineers is free (<http://i-concrete.epfl.ch>) and the source code of the program can also be downloaded and modified.

3. APPLICATION OF THE EPSF METHOD

3.1. Strength prediction

In this paper the results of the analysis with EPSF method of a total of 125 specimens of which 95 reinforced concrete beams and 30 dapped end beams are included. The tests refer to references (9) (10) (11) (12) (13) (14) (15) (16) (17) (18) (19) (20) (21) (22), with failures occurring in shear and due to detailing issues (anchorage capacity). The results (in terms of measured-to-predicted strength) are presented in Table 1.

Table 1. Statistical analysis of the results from the analysis of specimens from literature with EPSF.

All specimens		V_{test}/V_{EPSF}
Rectangular beams	# specimens included	39
	Average	1.02
	COV, %	8.08
T-beams	# specimens included	28
	Average	1.15
	COV, %	9.35
Prestensioned beams	# specimens included	28
	Average	1.08
	COV, %	8.26
Dapped end beams	# specimens included	30
	Average	1.00
	COV, %	10.5

In the included beam specimens, various parameters were investigated including the compressive concrete strength f_c , shear reinforcement ratio $A_s/b_{w,s}$, the shear span, type of load, amount and type of prestressing, cross section type and size. Some of the beams were tested in order to investigate the interaction between a support zone and anchorage zone. For all beams, high amount of longitudinal reinforcement was provided at the span in order to avoid bending failure. The ratio of the obtained in the test to the predicted load bearing capacity for the beams included in this paper as a function

of the compressive strength and amount of shear reinforcement are presented in Figure 4. In this database only beams, for which some anchorage of the longitudinal reinforcement beyond the support was provided, were included.

In most test campaigns for dapped end beams the main goal of the research is to investigate a particular reinforcement detail at the corbel, focusing on the vertical and inclined hanger reinforcement at the corner and the main horizontal reinforcement at the corbel. Another point of interest for researchers is the anchorage of the longitudinal reinforcement in the full depth section, which allowed also for the validation of the EPSF method for the modelling of anchorage of reinforcement.

3.2. Deformation capacity/Strain field

In Figure 5 specimen TA6 tested by Leonhardt (9) and the solution obtained with the EPSF method are presented and compared (measured-to-predicted load ratio equal to 0.96). The specimen was subjected to a two-point load. Due to symmetry conditions, only half of it was thus analysed. During testing, the authors measured the strains at different load stages and calculated the stresses in the stirrups using the experimentally obtained curves from the material tests. Figures 5e and 5f present the comparison between the measured and calculated with the EPSF method strains and stresses in the centre of the stirrups.

The authors took measurements on both side of the beam so the values plotted in the figure are the mean values of the two corresponding symmetrical measurements. By neglecting the tensile strength of the concrete, for low load levels the method overestimates the contribution of the stirrups to the shear resistance of the beam. However, as the load increases a very good agreement between measured and calculated strains and stresses can be observed. In addition, the method tends to underestimate the stiffness in the uncracked stage of the specimens. However, as the load increases the method can accurately predict the stiffness of the specimens and their deformation (1). The failure load is thus accurately predicted (failure developing after cracking stabilized).

3.3. Mode of failure

An important property of a good mechanical model is that it can predict not only the ultimate load but also the failure

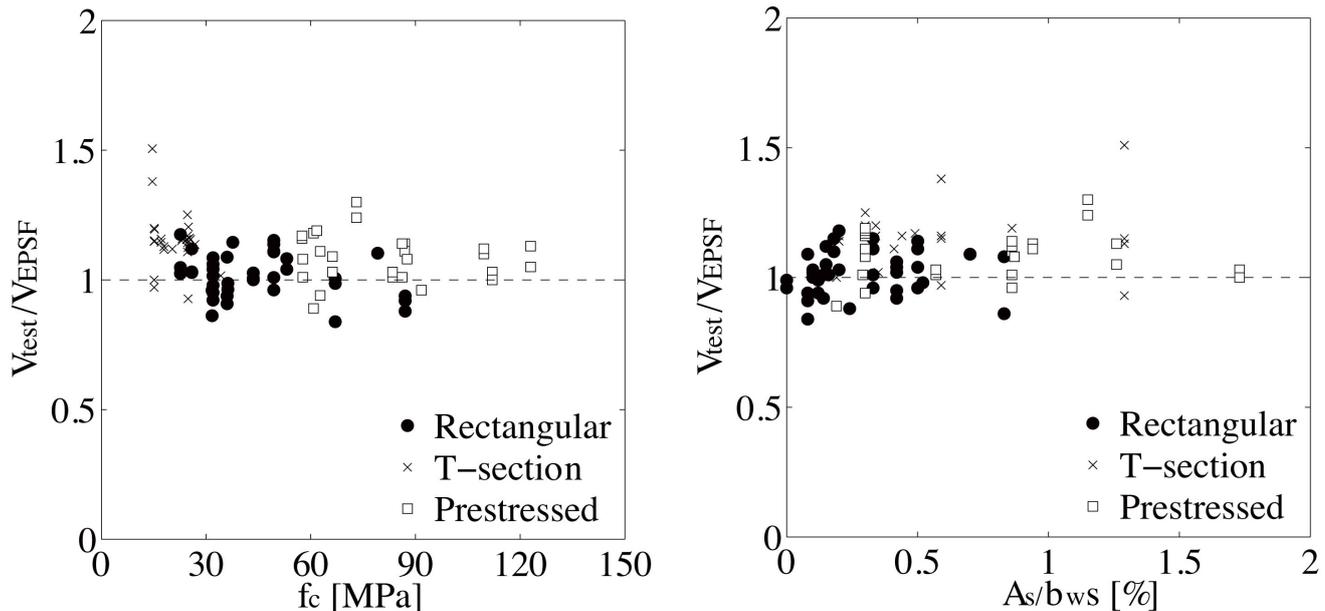


Figure 4. Results for the beams as a function of the (a) compressive strength; and (b) amount of shear reinforcement.

mode of an element. For the example presented in Figure 5 the authors reported that the failure occurred due to crushing of the web and yielding of stirrups. It can be clearly seen on Figure 5b and 5c that at failure the EPSF method was able to predict the proper mode of shear failure combining yielding of the stirrups and crushing of the concrete. It can also be observed that the zone of local crushing, predicted by the method, is the same as the one observed in the test Figure 5a. The analysis of zones with high shear forces is additionally complicated if a geometrical discontinuity is present. A good example is the dapped end beams. In the next paragraphs three common failure modes for dapped end beams and their EPSF solutions will be discussed.

The first failure mode is due to bending of the recessed end Figure 6a. As it can be observed in the example no extensive cracking or crushing of the concrete occurred at failure. The reinforcement had yielded at the wide cracked propagating form the re-entrant corner. The same mode of failure can be observed in the EPSF solution— the entire reinforcement at the re-entrant is yielding (Figure 6b) and the concrete has not reached its ultimate strength at the ultimate load (Figure 6c).

The second failure mode is due to a diagonal crack propagating from the bottom of the full depth section. During the testing of specimen DB1N, presented in Figure 7a, diagonal cracks, related to compression stress formed at the end of the full depth section. The crack across the bottom reinforcement formed at failure and opened to 6 mm. The authors concluded that failure occurred due to combined diagonal tension and bond failure. In the EPSF model the transfer length of the bottom reinforcement, which was not anchored by welded plates, was introduced by reducing the yielding stress of the bar elements and creating a linearly increasing load carrying capacity of the bottom reinforcement. The transfer length (l_b) was calculated as:

$$l_b = \frac{f_y \cdot \phi}{4 \cdot f_b} \quad [5]$$

where f_y is the yielding stress of the reinforcement, ϕ is the bar diameter, and f_b is the bond stress, which was assumed to be equal to twice the tensile strength of the concrete (for T-headed bars, the anchorage plate of the specimens was modelled according to its actual dimensions). It can be clearly recognized in the EPSF solution that at failure the load bearing capacity of the reinforcement in the bottom node was reached and that big tensile strains were obtained close to the failure region, which reduced the coefficient k_c and caused failure of the concrete elements. This solution clearly determines the exact cause of failure and the ultimate load bearing capacity of the specimen.

The third failure mode which will be discussed in this section is due to crushing of the concrete in the compression zone as it is for the example in Figure 8a. It is clear that no extensive cracking and crushing occurred in the full depth section and sufficient longitudinal reinforcement was provided in this part of the specimen. A free-body assumed from the stress distribution in the EPSF solution was isolated and from a simple calculation it was concluded that the specimen did not fail at the corbel. From the EPSF analysis it can be observed that as the deformation of the specimen increases so does the principal tensile strains at the corbel. This causes coefficient k_c to reduce and thus the compressive strength, leading to failure. This failure mode corresponds well with the observed one from the test.

4. CONCLUSIONS

This paper shows that for advanced modelling of a reinforced concrete member different strategies may be followed: use of sophisticated constitutive laws or use of simple constitutive laws. Both strategies are possible and have their advantages. For practical purposes, the use of simple constitutive laws seems however more suitable. This is justified as it is consistent to the design basis of simpler procedures, understandable by designers, keeps the same safety format and (provided that the analysis tool is used within its domain of application) yields results with the same level of accuracy as other more

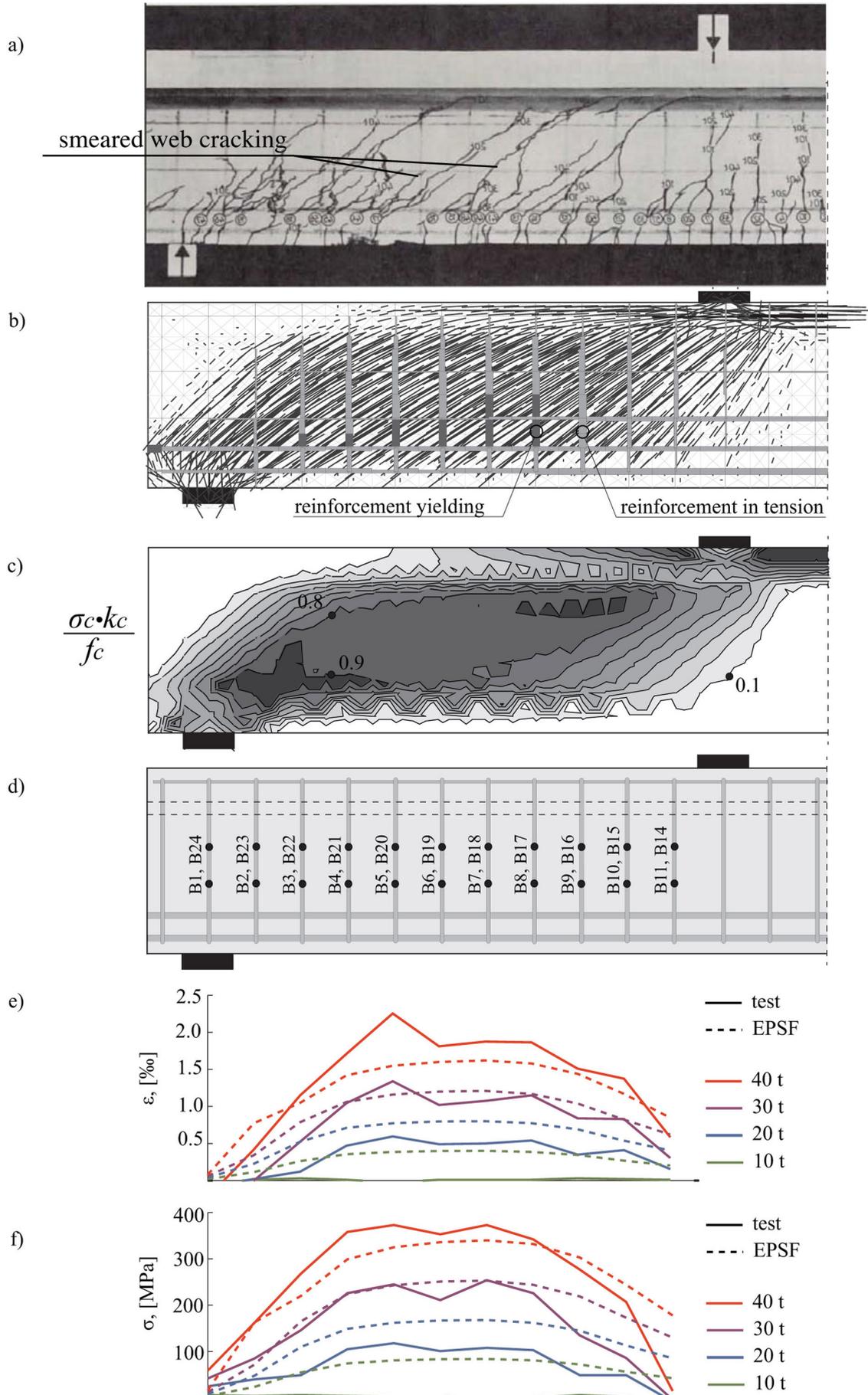


Figure 5. Beam TA6, (a) failure mode; (b) EPSF solution (c) degree of use of concrete strength (d) position and name of the measurements of the stirrups (e) strains in the stirrups, (f) stresses in the stirrups.

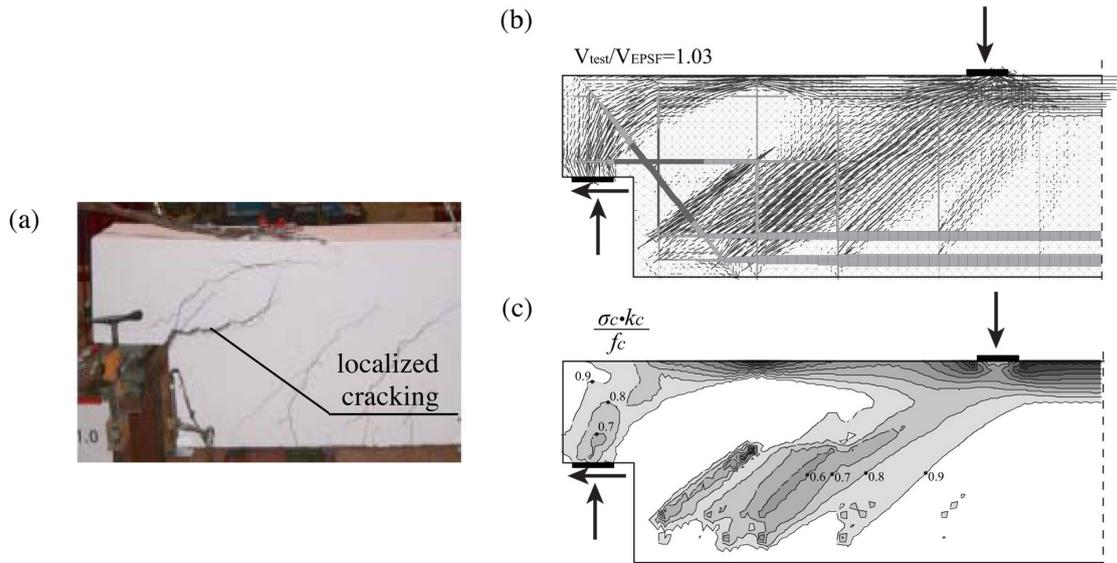


Figure 6. Failure mode with bending of the dap (a) specimen DE-D*(18) (b) EPSF solution, and (c) degree of use of concrete strength.

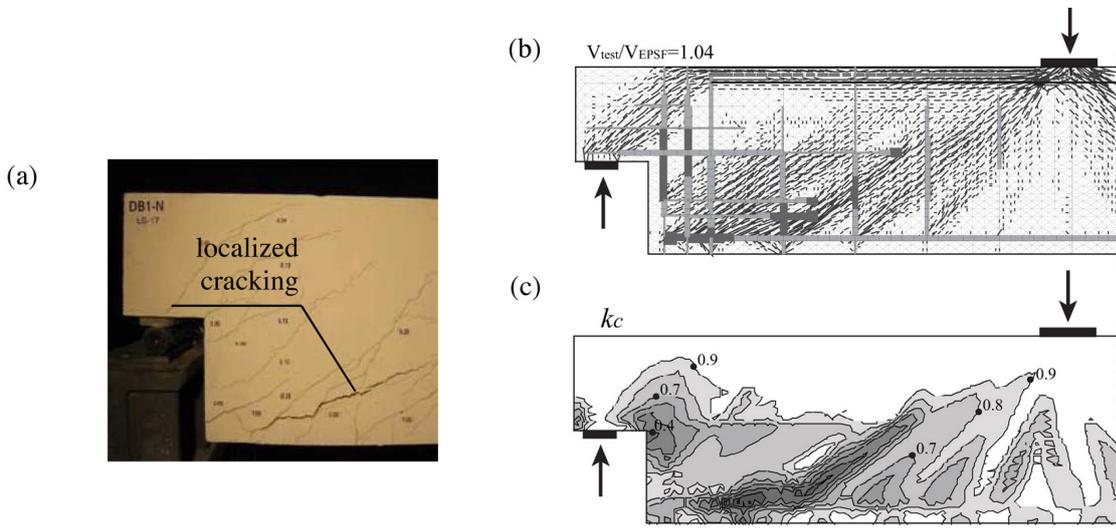


Figure 7. Failure mode with bending of the dap: (a) specimen DB1N; (b) EPSF solution; and (c) distribution of coefficient k_c .

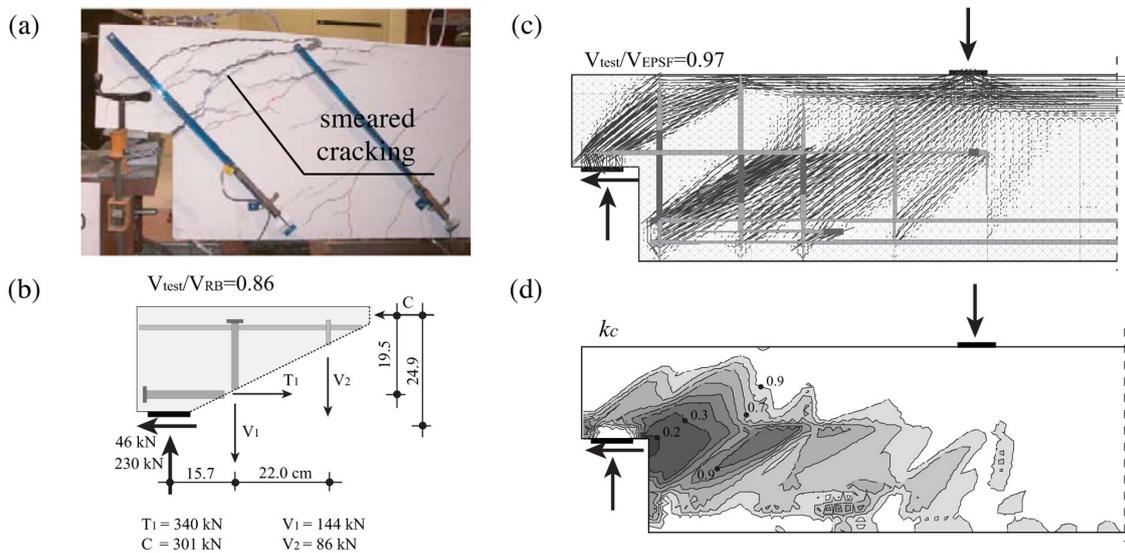


Figure 8. Failure mode in the compression zone: (a) specimen DE-B-1 (18); (b) rigid body at the dap; (c) EPSF solution; and (d) distribution of coefficient k_c .

sophisticated (yet more complex and more difficult to use) modelling strategies.

To this purpose, and for members with sufficient transverse reinforcement, this paper presents the main assumptions and implementation of the Elastic-Plastic Stress Field (EPSF) method and the results from the analysis of 125 tests from the literature. It is demonstrated by a number of examples that the method can provide a good estimate of the ultimate load

bearing capacity and deformations, as well as the mode and region of failure.

Therefore it can be concluded that the good level of accuracy of the EPSF method makes it a promising tool for the analysis of existing structures for engineers and researchers. Also, its simple and physical hypotheses encourage the use of such method for educational purposes.

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