Informes de la Construcción
Vol. 75, 570, e499
abril-junio 2023
ISSN-L: 0020-0883
https://doi.org/10.3989/ic.6175

Load distribution in flat reciprocal structures

Distribución de cargas en estructuras recíprocas planas

Laura Gonzalo-Calderón (*), José Ramón Aira (**)

ABSTRACT

The elements in conventional structures are perfectly ranked, so that load transmission is logical and follows the usual structural orders. Nevertheless, in reciprocal structures each element has to support all of the others in a less intuitive pattern of load transmission. The purpose of this paper is to understand exactly how load is transmitted between elements, quantifying this analytically by developing a new method which is applicable to a flat structure composed of a basic unit with any number of nexors. It is based on determining the increase in load to which the members in a reciprocal structure are subjected by calculating the coefficient *k,* or "transference coefficient". The *k* coefficient value, and therefore the load transferred between members, falls with the number of nexors, with the proximity of point loads to exterior supports, and with the size of the central space in the structure.

Keywords: reciprocal structures; timber construction; analytical method; load transmission.

RESUMEN

En las estructuras convencionales la transmisión de cargas es lógica y sigue los órdenes estructurales habituales. Sin embargo, en las estructuras recíprocas cada elemento tiene que soportar a todos los demás en un patrón de transmisión de cargas menos intuitivo. El objetivo de este trabajo es comprender exactamente cómo se transmite la carga entre los elementos, cuantificándolo analíticamente mediante el desarrollo de un nuevo método que es aplicable a estructuras planas compuestas por una unidad básica con cualquier número de nexors. Se basa en la determinación del incremento de carga al que están sometidos los miembros de la estructura recíproca mediante el cálculo del coeficiente k, o "coeficiente de transferencia". El valor del coeficiente k, y por tanto la carga transferida entre los miembros, disminuye con el número de nexors, con la proximidad de las cargas puntuales a los apoyos exteriores y con el tamaño del espacio central.

Palabras clave*: estructuras recíprocas; construcción con madera; método analítico; transmisión de cargas.*

(*) Arquitecta. Profesora Asociada. Departamento de Estructuras y Física de Edificación. Escuela Técnica Superior de Arquitectura. Universidad Politécnica de Madrid (España).
(**) Doctor Ingeniero de Montes. Profesor Contratado Doctor. Departamento de Construcción y Tecnología Arquitectónicas. Escuela Técnica Superior de Arquitectura. Universidad Politécnica de Madrid (España).
Persona de contacto/*Corresponding author*: joseramon.aira@upm.es (J.R. Aira)
ORCID: http://orcid.org/0000-0002-0105-5850 (L. Gonzalo-Calderón); http://orcid.org/0000-0002-4598-5259 (J.R. Aira)

Cómo citar este artículo/*Citation:* Laura Gonzalo-Calderón, José Ramón Aira-Zunzunegui (2023). Load distribution in flat reciprocal structures. *Informes de la Construcción,* 75(570): e499. https://doi.org/10.3989/ic.6175

Recibido/*Received*: 11/06/2022
Aceptado/*Accepted*: 13/02/2023
Publicado on-line/*Published on-line*: 14/06/2023

1. INTRODUCTION

Reciprocal structures have been used for centuries because they are an ingenious solution when the aim is to roof large spans using short elements and engagement lengths (1-3). The simplest forms of this type of structure are both temporary and reversible, and they are made using elements that are highly transportable.

Wood is a perfect material for building reciprocal structures as it is very resistant against forces parallel to its fibre, very low in weight in comparison with its strength, flexible so that elements can be joined without the need to use heavy mechanical aids, and with good elastic capacity so that it can be assembled and disassembled when necessary.

As is the case in other types of ancient constructions, the basic material consisted of small pieces of wood from nearby forests, as this was the most economical solution, giving immediately available components.

According to del Rio (4) the oldest texts documenting reciprocal structures date back to the Middle Ages with Villard de Honnecourt (5) and early Renaissance with Leonardo da Vinci and Sebastian Serlio (6-8). Indeed, Leonardo da Vinci made several sketches of floors and roofs to cover surfaces and even as bridges to pass over linear obstacles such as rivers, roads, etc. (9), Figure 1. An in-depth review of the scientific literature on reciprocal structures may be found in previous work carried out by other researchers (10, 11).

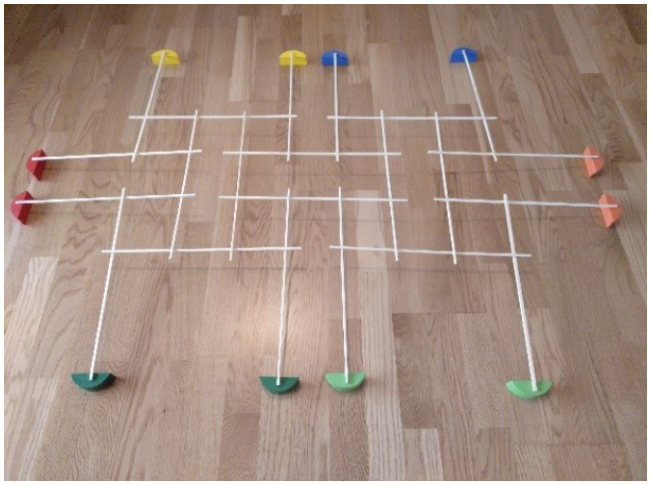

Figure 1. Scale model of Leonardo da Vinci's sketch.

Although this is not a very common structural typology, reciprocal structures fell into total disuse after the second half of the 18th century because the industrial revolution brought about the development of new structural materials and the production of longer elements able to cover long spans.

However, in recent decades this type of structure has found a new market niche in what is termed ephemeral architecture. This consists of temporary works that are fully reusable, fitting very well with the current tendencies for sustainable architecture or bioconstruction. The scientific literature contains several examples of three-dimensional reciprocal constructions which express this resurgence (12-16). Another advantage of this type of structure is that prefabricated parts can be used, with simple or complex geometry, which are easy to assemble on-site (17).

It is important to use the same terms when referring to the elements of these structures when studying them, Figure 2. Reciprocal structures are also known as "nexorades". The term "nexor" stems from Latin, and it means "nexus" or "connection", so that the term "nexorade" means "assembly of connections".

Figure 2. Basic units, or fans, of 3 nexors and 4 nexors.

Reciprocal structures or nexorades are modular, and they are composed of basic units denominated "fans". Each fan is composed of at least 3 nexors, which is the term used to refer to the members in a fan (18).

Depending on the arrangement of these basic units or fans, nexorades can be of three types: "simple" when the structure consists of a single basic unit of either 3 nexors, 4 nexors, 5 nexors, etc.; "multiple" when the structure consists of a combination of several basic units, such as a combination of several basic units of 3 nexors with several basic units of 4 nexors; and "complex" when the structure consists of the extension by repetition of the same basic unit (11).

A regular structure is obtained when the members of a reciprocal structure are arranged regularly around a central point of symmetry. An irregular structure results from the repetition of basic units in a disorganised way without a regular pattern (19).

2. JUSTIFICATION AND OBJECTIVES

In recent years reciprocal structures have been increasingly used by engineers and architects, due to their high degree of congruity with certain current trends. Thus some researchers undertook the laborious task of collecting the most important design aspects used to date, with the aim of encouraging their use in architecture (11). They stated that in conventional structures the challenge is to adapt structure to architectural design requirements. However, in reciprocal structures the challenge arises in the design phase, due to the large number of variables that influence structural shape and behaviour.

Some researchers have concentrated on the study of reciprocal structures from a geometrical point-of-view, establishing and optimising the parameters that determine their shape (18, 20-25). Although other researchers have tried to explain how these structures work in mechanical terms, it has yet to be explained in depth how loads are transmitted between elements, i.e., exactly what the reciprocal behaviour of structures of this type consists of.

The elements in conventional structures are ranked perfectly, so that load transmission is logical and perfectly follows the structural orders set by the design. Nevertheless, in a recipro-

cal structure each element in it has to support all of the others to a greater or lesser degree, in a non-intuitive pattern of load transmissions between them. The principle of reciprocity is defined as the use of supporting elements which, resting on each other, form a spatial configuration without any clear structural hierarchy (10).

In a reciprocal structure, no substructure is stable until the complete structure has been assembled, and moreover if one of its elements is eliminated, then each part of the structure will be mobile respecting all of the other parts, i.e., this would create a mechanism (26). The work by John Wallis in his *Opera mathematica* stands out in this respect (27). This was recalled and examined by Houlsby (26). Wallis analysed the distribution of loads in a reciprocal structure that had been proposed by Leonardo da Vinci. This is a flat reciprocal structure floor created by repeating a 4 nexor basic unit. To do this Wallis only considered the load arising from the weight of the elements themselves. By establishing the balance of moments of each member, he obtained 25 linear equations (first-degree equations) that resolved the 25 unknowns or loads at the connections in the structure. Wallis's working method of dividing the problem into simple interconnected parts was brilliant, and there can be no doubt that it was the predecessor of the finite elements technique now in widespread use for structural analysis. However, Wallis's analysis centres on a particular reciprocal structure and also uses a type of load (self weight) that generates no load-share asymmetries in the supports. Thus, although this approach hints at the presence of reciprocal behaviour, this is never made explicit, and moreover it is not possible to extrapolate its results to reciprocal structures that differ from the one analysed.

Gelez analysed the behaviour of reciprocal structures using a method that is half analytical and half numerical, to obtain the maximum moment and the deformation at the centre of a basic unit (28). The calculation hypothesis used load that was distributed over the entire surface, simulating a real load state. Nevertheless, this consideration made it impossible to find how loads were actually transmitted between the different elements, as equal reactions are obtained in all of the exterior supports of the structure. They also reached several relevant conclusions regarding its structural behaviour. They defined the difference between reciprocal structures and flat meshes on the basis of their degrees of freedom, indicating that flat meshes prevent turning around the strong axis of the section, which does not occur in reciprocal structures. Given this consideration, in the centre of reciprocal structures greater bending moments and deformations arose than was the case in flat meshes, which is logical as flat meshes have more rigid connections that transmit the load to supports around the edge, instead of being supported by elements within the structure itself. They also stated that reciprocal structures are less robust than flat meshes. This is because reciprocal structures are considered to be statically determined, i.e., load is distributed by a single transmission mechanism, and if any one of its elements is lacking then it collapses. On the other hand, in a flat mesh there are different load transmission mechanisms in case any element is missing.

The fact that a flat reciprocal structure is considered to be statically determined indicates that it is not necessary to undertake deformation compatibility analysis to discover the unknowns in the system. The loads which act on each connection will therefore be independent of the rigidity of the material and member cross-section.

Kohlhammer and Kotnik used a repetitive approach to discover the distribution of loads among connections, under the action of the self weight of the elements in a flat reciprocal structure (29). They correctly considered that the load acting on each connection varies in each one of the repetitions. Each repetition consisted of distributing the loads of the members which received load from the members in contact with them. The loads in the connections were calculated computationally by considering a static balance of the system after each repetition. Finally, to verify their methodology, they compared the results with a finite element analysis, finding very slight differences between them.

The aim of this research work consists of understanding exactly how load transmission occurs between the members of a simple reciprocal structure and obtaining, analytically, the amount of this load in each point of the system. Once the exact amount of load on each member of a reciprocal structure and the influence of the geometry on the load transmission is known, it will be possible to design such structures more accurately and more quickly. The analytical equations obtained will be used as the basis for further research work that analyses more complex reciprocal structures.

3. MATERIALS AND METHODS

The methodology consists of analysing how loads are transmitted between the members of a flat reciprocal structure composed of a basic unit of *n* nexors.

The structure is considered to be subjected to perpendicular exterior loads, while ignoring the self weight of the elements. Point loads as well as loads distributed over members are analysed at any position in the structure.

The initial hypothesis considers a reciprocal structure with a point load applied on one of its members. The static balance of the member that receives the load is established, and so on successively until the final member is reached, observing that a part of the initial load returns to the first member after passing through all of the members in the structure. With this the static balance of the first member is lost, so that the extra load has to be distributed once again among all of the members. In each repetition the extra load is less each time, in a process that gradually approaches zero. The calculation methods proposed by some researchers are based on this consideration (2, 29).

Based on the above argument it is possible to conclude that every time an exterior load is applied to a member in a reciprocal structure of the type studied, this will be distributed between an exterior support and an interior support, and that, in turn, the load that reaches the inner support must be supported once again by the whole structure, including the member that receives the exterior load.

The method described in this work consists of firstly establishing this increase in load on the first member in order to make the distributions to the others only once, i.e. only the final state of static equilibrium is considered. The value of the transferred load is unknown but it is clear that it will be

directly proportional to the load applied, P, so it will take a value of kP, where k is called the "transfer coefficient" that the method aims to determine. The k value must be such that the balance of the whole structure can be established in a single round, without the need for iterative processes and with mathematically exact results.

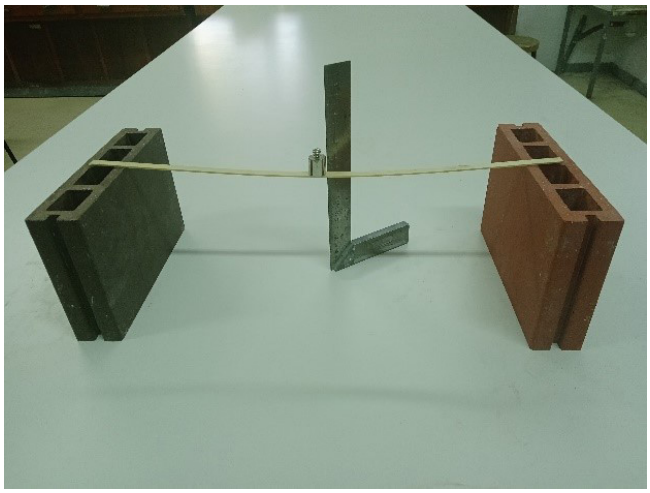

Figure 3. Vertical deflection in the centre of the span of a simple member.

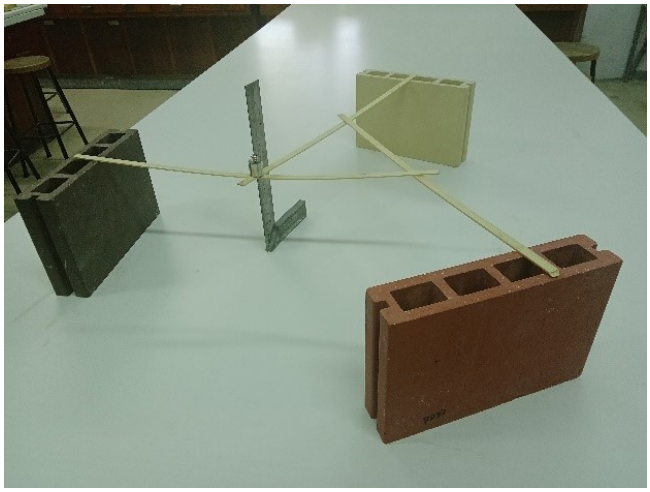

Figure 4. Vertical deflection in the centre of the span of member into reciprocal structure.

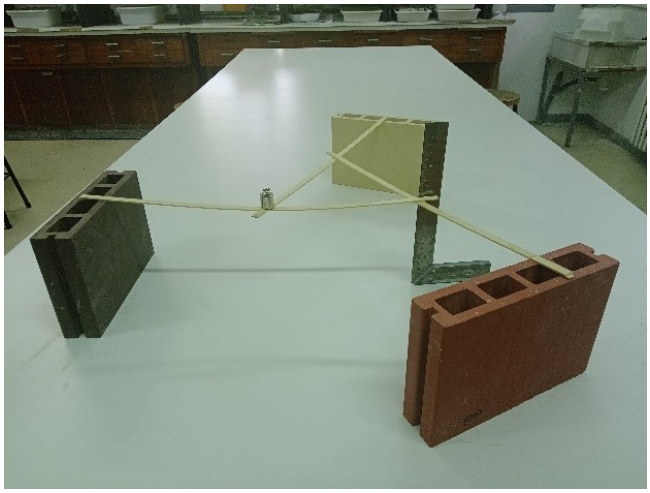

Figure 5. Vertical deflection at the end of member into reciprocal structure.

The study takes place in 3 phases. Phase 1 considers the action of point exterior load applied to a reciprocal structure composed of a basic unit of 3 nexors. In phase 1 a generic equation is obtained to calculate the transfer coefficient, k, which makes it possible to know the loads which act on the members for any point load situation. Phase 2 considers the action of uniformly distributed exterior load on the same reciprocal structure. In the same way, in phase 2 another generic analytical equation is obtained to calculate the transfer coefficient, k, for situations involving a uniformly distributed exterior load. In phase 3, an application example is shown for a more complex reciprocal structure composed of a 7 nexor basic unit, with a point load on one of them and a uniformly distributed load on another. The equations are subsequently extrapolated to a structure of n nexors.

The results obtained analytically in phases 1, 2 and 3 are compared with the results shown by calculation software to verify the validity of the method. The verification of the analytical equations obtained is carried out using the Dlubal© software, version 5.21.02. For member-type elements (1D), this software uses the Matrix Stiffness Method which proposes a final equilibrium of the system to obtain the internal forces and reactions from the displacement of nodes.

In the end, an experimental demostration is carried out for a basic unit of 3 nexors. For this purpose, a 10-gram weight is placed in the centre of a simple timber member of cross-section 1x10 mm and span 420 mm, and the vertical deflection in the centre of the span is measured, Figure 3. The same operation is repeated with a 20-gram weight. As the loads are small, the wood is in an elastic state and Hooke's law is verified, i.e. the deflection is proportional to the applied load. Subsequently, the member is assembled together with 2 others (with same properties and dimensions) to make a basic unit of 3 nexors. Again, a 10-gram weight is applied and the vertical deflection is measured at the centre of the span and at the end, Figures 4 and 5. To calculate the effective deflection at the centre of the span, half of the end deflection is subtracted. The same operation is performed with a 20 gram weight.

This experimental verification consists of checking that the effective deflection suffered in the centre of the span by the member into reciprocal structure is the same as suffered when it is alone, but multiplied by $1 + k$.

4. RESULTS AND DISCUSSION

4.1. Phase 1: Point load

Firstly, study centres on how a point load P is transmitted between the elements of a basic reciprocal structure composed of 3 members (or 3 nexors) L in length, joined together at their central points and with articulated supports at their exterior ends, placing the load at the intersections between two members, Figure 6. The load is applied at the centre of member 1, where, additionally, it is joined to member 2.

On the loaded member, a is the distance from the support to the point of application of the load, and b is the rest of the member ($b = L\text{-}a$). In this case, $a = b = L/2$. In all of the

members *c* is the distance from the support to the intersection with another member, and *d* is the distance between the intersections with both members (*d* = *L*-*c*). In this particular case, *c* = *d* = *L*/2.

As was pointed out above, the most intuitive way of approaching the problem would firstly involve considering that the load *P* on member 1 is distributed between its two ends, so that it transfers *P*/2 to the exterior support of the member, and *P*/2 to the next member, member 2. Member 2 would be loaded at its centre point with a load *P*/2, that would be distributed between the support and the connection with member 3, each one of which would take *P*/4. This load *P*/4 on the central point of member 3 would be distributed between its support and the connection with member 1, with a value of *P*/8. This load transfer, which returns to the point of application after passing through all of the members, would lead to member 1 losing the static balance that had been defined initially, so it would be necessary to repeat the same process for this load, *P*/8. It may therefore be deduced that the distribution of loads in the structure would be an iterative process in which it would only be possible to achieve an approximate result, aiming for a value close to 0 for the "rest" of the undistributed load, although never equal to 0 from a strictly mathematical point of view.

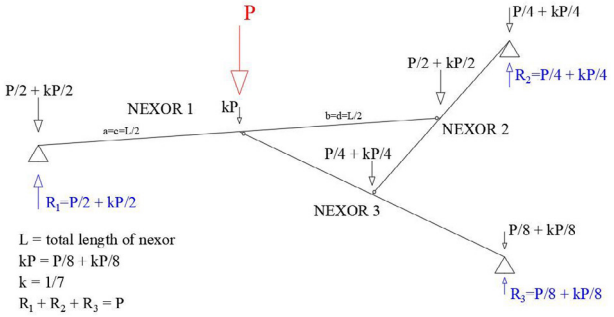

Figure 6. Basic unit of 3 nexors with P load centred on a member.

Applying the proposed method, the load at the centre of member 1 will be *P*+*kP*, which is symmetrically distributed between the exterior support and the intersection with member 2, *P*/2+*kP*/2. In member 2, both the exterior support and the intersection with member 3 will receive half of this load, *P*/4+*kP*/4, which in turn will transfer to its exterior support and the intersection with member 1, taking the value *P*/8+*kP*/8 to both ends. For the system to be in balance, the load deposited by member 3 in member 1 must be exactly the transferred load, *kP*. Finally, considering the static equilibrium, *kP* = *P*/8+*kP*/8, it is concluded that the transfer coefficient for this configuration has a value of 1/7.

To verify whether the approach is correct, this structure was modelled using the finite element software. In this, in a basic unit of 3 1000 mm members joined at their mid-points, a load of 1 N is placed at the centre of member 1, Figure 7.

The shear graph shows that the load applied by member 3, which is on member 1 at the point of application of the same, is 0.143 N, i.e., 1/7 *P*. This means that the reaction on the first support will be 0.571 N, that is, *P*/2+*kP*/2. On the following members, the load reaching them in each case is distributed

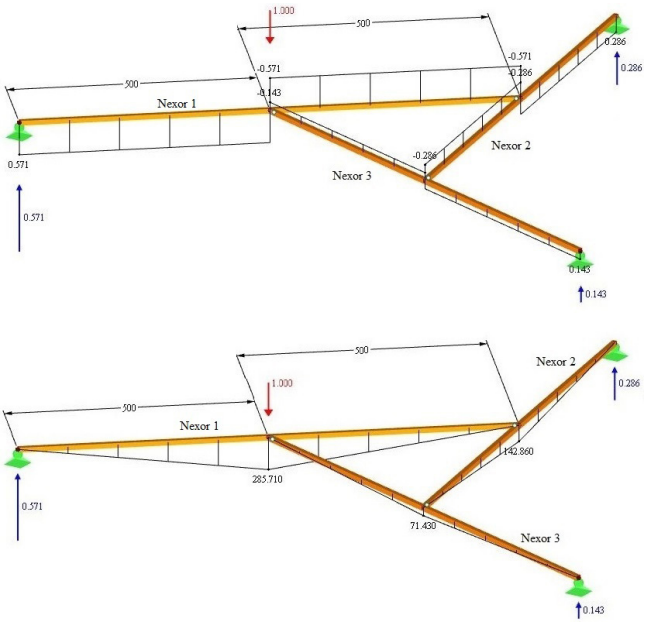

Figure 7. Shear and bending moment distribution in a basic unit of 3 nexors connected by their midpoints, with point load centred on member 1 (external loads [N], support reactions [N], internal forces Vz [N] and My [Nmm], nexor length [mm]).

equally between the exterior support and the interior support on the next member, until the final distribution which leaves 1/7 on member 1. In this way, the load has been completely distributed in the structure without the need for an iterative process to discover how closely it approximates to reality.

The moments graph shows that the members are articulated at their exterior support as well as the intersection with the next member, and that in each of them the moment would correspond to an isostatic beam with two supports that supports the loads found by means of the previous procedure. We can therefore see that once the loads have been found, analysis of thestructure is simple and that there is no need to undertake deformation compatibility.

To verify the dependency of the transfer coefficient *k* on the position of the load and the point at which the members intersect, the load is now placed on member 1 in any position, i.e., with parametric values of *a* and *c*, Figure 8. It should be pointed out that it is irrelevant whether the load *P* is located in the section of dimension *c* or in the interior polygon of the basic unit.

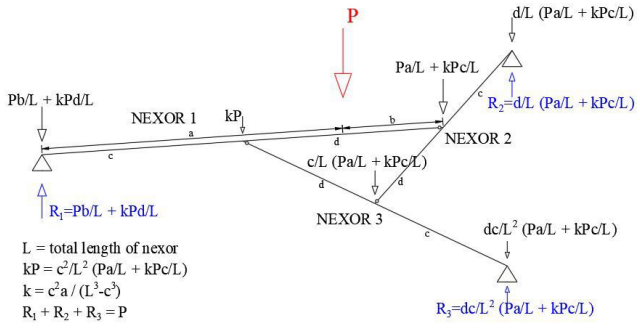

Figure 8. *Basic unit of 3 nexors with P load in any position.*

P and the transferred load are now not applied at the same point. According to the principle of load superposition, the load which reaches each end of the member is the sum of the load corresponding to it for each one of the loads. Thus at the exterior support of member 1 Pb/L and kPd/L are obtained, while at the intersection with member 2, Pa/L and kPc/L are obtained.

After this point the load distributions no longer depend on *a* and *b*, but rather depend exclusively on *c* and *d*, which are the same on all of the members. Each load applied therefore leads to a reaction in the exterior support equal to the said load, by d/L, and on the next member, with a load by c/L. In this case, in which the basic unit has 3 members, the load which returns to the member is equal to $Pa/L+kPc/L$ multiplied twice by c/L, once for transmission from member 1 to member 2, and once for transmission from member 2 to member 3.

Proceeding in the same way as in the first example, kP is equal to the load obtained at this point, i.e., $kP = c^2/L^2(Pa/L+kPc/L)$, where the transmission coefficient is obtained, $k = c^2a/(L^3-c^3)$. The value of *k* is adimensional and depends on the c/L and a/L ratio.

The software is used again to perform the verification, but with a load that is not centred. Using the previous model, the load *P* of 1 N is displaced in such a way that $a = 3L/4$. The dimensions of the structure remain the same, so that $c = L/2$, Figure 9.

In this case, the shear graph shows that the load transferred by member 3 over member 1 is 0.214 N. If the equation $k = c^2a/(L^3-c^3)$ is applied, this gives $k = 0.214$. The reactions in the supports can also be checked, for example by taking the equation $R_2 = d/L(Pa/L+kPc/L)$ from Figure 8, giving $R_2 = 0.429$ N, which fits the value shown by the software.

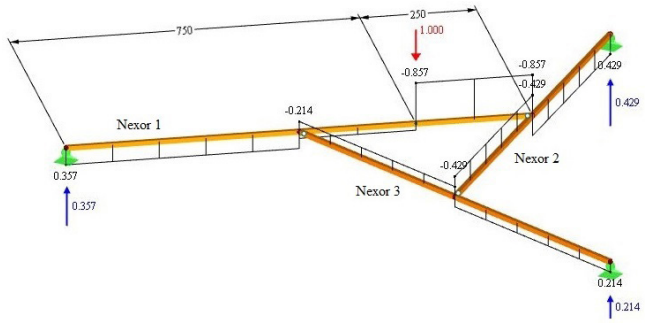

Figure 9. Shear distribution in a basic unit of 3 nexors connected by their midpoints, with point load not centred on member 1 (external loads [N], support reactions [N], internal forces Vz [N], nexor length [mm]).

Although work took place with basic 3-member units (or 3 nexors), the process would continue in the same way for a greater number of members, multiplying by d/L in the exterior support and by c/L in the interior support on the next member. It is therefore possible to formulate the value of *k* for a generic case with *n* members, based on the load distribution pattern for member *n*, Figure 10.

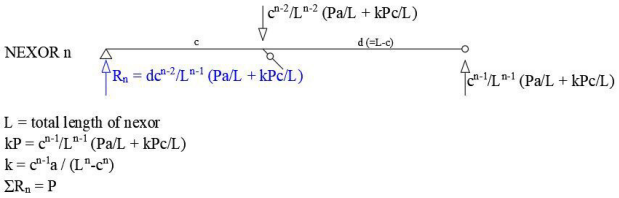

Figure 10. *Load distribution on member n for a basic unit of n nexors.*

In a reciprocal structure composed of a basic unit of *n* nexors of length *L*, subjected to a point load *P* in any position, the transferred load which returns to the first member is equal to an adimensional coefficient *k* multiplied by the load *P* applied, according to the expression [1].

$$[1] \quad k = \frac{c^{n-1}a}{L^n - c^n}$$

where *k* is the transfer coefficient, *n* the number of nexors, *L* the nexor length, *a* the distance of the load from the exterior support of the member on which it is applied, and *c* the distance from the intersection between members to the exterior support.

4.2. Phase 2: Uniformly distributed load

In structures of this type it is habitual for distributed loads to appear. These usually arise due to the working load considered by regulations, the self weight of the members comprising the structure, or other elements that weigh on the same. Due to this, the way uniformly distributed load is transmitted in a reciprocal structure is analysed below.

To return to the basic 3-member (or 3 nexors) unit, a uniformly distributed load *q* is considered on member 1, Figure 11. The transferred load which returns to member 1 will on this occasion be kqL, as it is a point load. On member 1, as *q* is a distributed load, it will be transmitted symmetrically between the exterior support and the interior support on member 2, $qL/2$ for each one. The corresponding distribution of load kqL will have to be added to this term, which in this case depends on *c*, where $kqLd/L$ is the load on the exterior support and $kqLc/L$ is the load on member 2, values that may be simplified as kqd and kqc, respectively. From here on the distribution is performed in exactly the same way as it was in phase 1, given that point loads are distributed to the other members. Completing the distribution and finding the value of *k*, it is observed that this responds to the expression $k = c^2L/2(L^3-c^3)$.

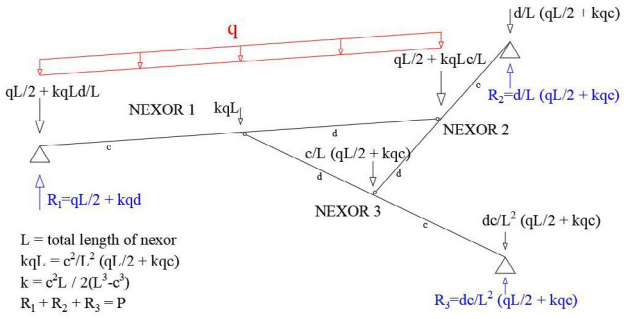

Figure 11. Basic unit of 3 nexors with uniformly distributed q-load.

To generalise, for a reciprocal structure composed of a basic unit of n nexor L in length, subjected to a uniformly distributed load q, the transferred load that returns to the first member is equal to an adimensional coefficient k multiplied by qL, based on the expression [2].

$$[2] \quad k = \frac{c^{n-1}L}{2(L^n - c^n)}$$

where k is the transfer coefficient, n the number of nexors, L the nexor length, and c the distance of the intersection between members and the exterior support.

In the same way, the analytically obtained value of k is compared with the value obtained using the software, but applying a uniformly distributed load of 1 N/mm on member 1. Dimensions of the structure remain the same, so that $c = L/2$, Figure 12.

Applying the analytical formula [2] to this particular case gives $k = 1/7$. The transferred load is therefore 142.857 N, as may be seen in the shear diagram.

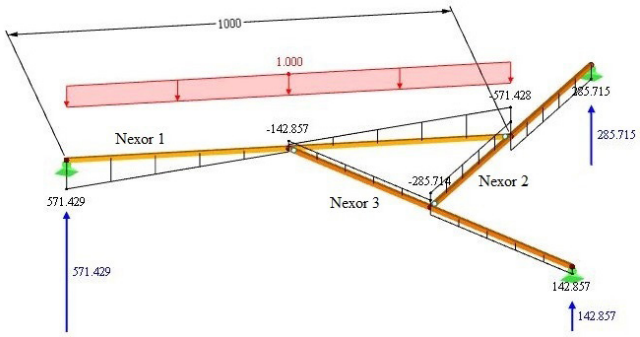

Figure 12. Shear distribution in a basic unit of 3 nexors connected by their midpoints, with uniformly distributed load on one of their members (external loads [N/mm], support reactions [N], internal forces Vz [N], nexor length [mm]).

4.3. Phase 3: Application example

To verify the validity of formulas (1) and (2) a more complex particular example was used, consisting of a reciprocal structure composed of a basic 7-member unit with generic distances c and d. A point load P is considered, applied at any position on one of the members, and another uniformly distributed load q applied on another one of the members, Figure 13.

The distribution of loads P and q in the structure is totally independent, so that the transfer coefficient corresponding to each load is then found. In each case this will make it possible to find the transferred load that returns to the member subjected to each one of the exterior loads.

The transfer coefficient k_1 refers to point load P, while k_2 is linked to load q. The load distribution throughout the structure is shown in figure 13, so that it is possible to analyse each one of the members based on the sums corresponding to both load states.

The graph shown in figure 14 is obtained by analysing the reactions in the exterior supports and shear distribution using the software. The conjoint action of a point load P of 2000 N applied on nexor 1 is considered, together with a distributed load q of 3 N/mm applied to nexor 3.

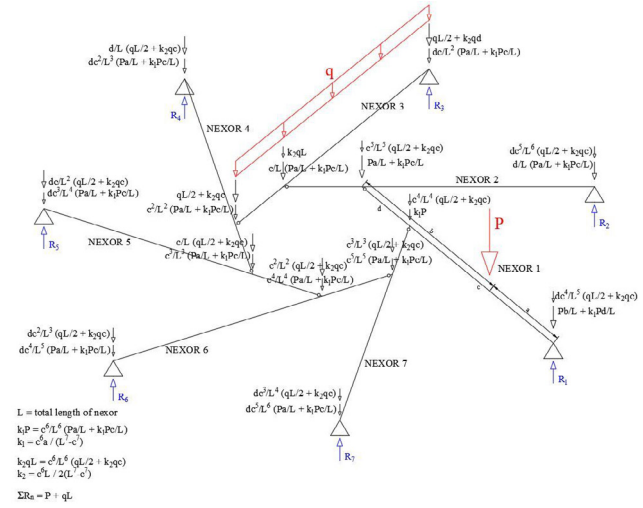

Figure 13. Basic unit consisting of 7 nexors with any point load and any uniformly distributed load.

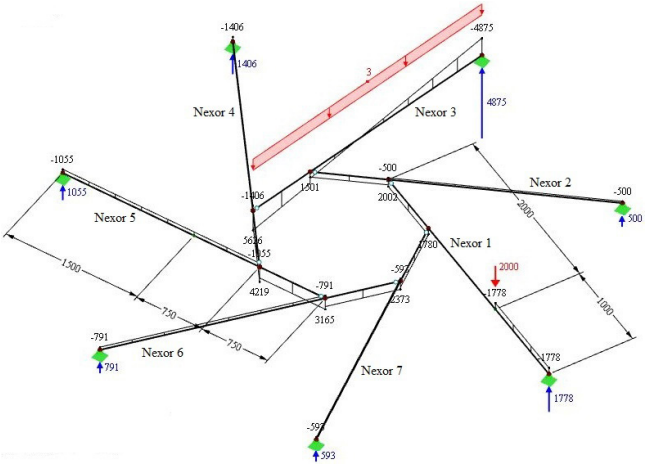

Figure 14. Shear distribution of a basic unit of 7 nexors with any point load and any uniformly distributed load (external loads [N] and [N/mm], support reactions [N], internal forces Vz [N], nexor length [mm]).

By applying equations [1] and [2] to this particular case the transfer coefficients are obtained, at $k_1 = 0.068$ and $k_2 = 0.103$, respectively, so that the transferred loads amount to $k_1P = 137$ N and $k_2qL = 924$ N. Based on these values it is possible to find the loads in all of the members with the parameter values shown in figure 13. Thus and for example, the rise in the shear graph in nexor 1 at its intersection with nexor 7 has an analytical value of $k_1P+c^4/L^4(qL/2+k_2qc) = 1780$ N, which corresponds with the value of the figure 14; the rise in the shear graph for nexor 6 at its intersection with nexor 5 has a value of $c^4/L^4(Pa/L+k_1Pc/L)+c^2/L^2(qL/2+k_2qc) = 3165$ N; and the reaction in the exterior support of nexor 4 has a value of $R_4 = dc^2L^3(Pa/L+k_1Pc/L)+d/L(qL/2+k_2qc) = 1406$ N.

4.4. Experimental verification

Table 1 shows the results of the experimental tests.

Applying equation [1] for the experimental test conditions, i.e. for $n = 3$ nexors, $L = 420$ mm, $a = 210$ mm and $c = 210$ mm; a transfer coefficient $k = 0.14$ is obtained. This value is

the same as the effective transfer coefficient experimentally obtained, thus proving the validity of the analytical equations.

Tabla 1. Experimental determination of the transfer coefficient in basic unit of 3 nexors with a load centred on a member.

	Measurement location	**Deflection 10-gram weight**	**Deflection 20-gram weight**
Simple member	centre of the span	20 mm	40 mm
Member into reciprocal structure	end of the span	14 mm	30 mm
	centre of the span	30 mm	60 mm
	centre of the span (effective)	30 – 14/2 = 23 mm	60 – 30/2 = 45 mm
Load increase between simple member and member into reciprocal structure		23/20 = 1.15	45/40 = 1.13
Transfer coefficent (*k*)		1.15 - 1 = 0.15	1.13 - 1 = 0.13
Average transfer coefficient (*k*)		0.14	

4.5. Discussion

The analytical results obtained using equations [1] and [2] are identical to those shown by the Matrix Stiffness Method software, thereby verifying the proposed method. In addition, the analytical equation [1] was also verified by a simple experimental test.

Based on equations [1] and [2], it can be said that the greater the value of *c*, the greater will be coefficient *k*. This means that the smaller the central hollow in the reciprocal structure, the greater will be the transferred load, so that designing structures with larger hollows makes it possible to reduce the load increase on each member.

The exterior supports closest to the exterior load receive a higher proportion of the same. From equation [1] it may be deduced that when the value of *a*, is reduced, i.e., when the load is positioned closer to the exterior support, coefficient *k* also falls, reducing the load transferred between members. This is because a large proportion of the exterior load is transmitted directly to the exterior support.

On the other hand, it can be deduced from the method itself that the greater *n* is, the lesser will be coefficient *k*, i.e. the transferred load will be distributed among a larger number of members before returning to the initial member. Due to this, in reciprocal structures with a high number of nexors, each one of them will be subjected to less load.

5. CONCLUSIONS

This research work describes the development of a new method which makes it possible to understand and analytically quantify how loads are distributed among the members and supports of a flat reciprocal structure.

It is based on determining the increase in load to which the members of a reciprocal structure are subjected by calculating coefficient *k* or the transfer coefficient. This is given by equations [1] and [2] for point loads and uniformly distributed loads, respectively. Moreover, the superposition principle means that this method can be used for any combination of different loads in the members.

The value of coefficient *k*, and therefore the load transferred between members, falls with increasing numbers of nexors, with increasing proximity of point loads to the exterior supports, and with increasing size of the central hollow in the structure.

The present research work is limited to the study of a basic unit consisting of any number of nexors of the same length and with central symmetry. This basic unit can be subjected to any system of loads, either point and/or distributed loads.

In structures with more than one basic unit (multiplex, complex, etc.) the reasoning used to determine the load transfer would be similar, but number of equations would increase considerably and will be the subject of future research.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to the Materials Laboratory of the School of Architecture of the Universidad Politécnica de Madrid for providing their facilities for this research, as well as to the company Dlubal Software GmbH for providing their software.

REFERENCIAS / REFERENCES

(1) Chilton, J.C.; Choo, B.C. 1992. Reciprocal frame long span structures. *Proceedings of the International Association for Shell and Spatial Structures*. Canadian Society of Civil Engineers. International Congress on Innovative Large Span Structures. Montreal, Canada. July, 2, pp. 100 - 109.
(2) Chilton, J.C.; Choo, B.S.; Yu, J. 1994. Morphology of Reciprocal Frame Three-Dimensional Grillage Structures. Spatial, Lattice and Tension Structures. *ASCE*. pp. 1065 - 1074.
(3) Aira, J.R.; Cabo-Fernández, M.C.; Blanco-García, F.; Gonzalo-Calderón, L. 2021. Puentes de madera con estructura recíproca. Análisis comparativo del puente tradicional chino y el puente de Leonardo. *Informes de la Construcción* Vol. 565. https://doi.org/10.3989/ic.85874.
(4) Del Río-Calleja, B.; Grau J.; García-Santos A. 2022. Architectural systemic approach: The Serpentine Gallery 2005, a reciprocal frame case study. *Buildings* 2022, 12 (7), 1051. https://doi.org/10.3390/buildings12071051.
(5) De Honnecourt, V. 1235. Albun de design et croquis. *Livre de Portraiture*. Ms. Fr. 19093. Dèpartment des manuscrits, Bibliotèque Nationale de France. Paris, France.
(6) Da Vinci, L. 1478-1519. Codex Atlanticus. Biblioteca Ambrosiana. Milano, Italy.
(7) Da Vinci, L. 1490-1508. Tratado de estatica y mecaniha. *Codex Madrid I*. Mss 8937. Biblioteca Nacional de España. Madrid, Spain.
(8) Serlio, S. 1551. Archiitettura di Sebastian Serlio. Venetis s.n. Venetia, Italy.

(9) Di Carlo, B. 2008. The wooden roofs of Leonardo and new structural research. *Nexus Network Journal* 10, pp. 27-38. https://doi.org/10.1007/ S00004-007-0054-X.
(10) Pugnale, A.; Sassone, M. 2014. Structural reciprocity: critical overview and promising research/design issues. *Nexus Network Journal* 16, pp. 9-35. https://doi.org/10.1007/s00004-014-0174-z.
(11) Asefi, M.; Bahremandi-Tolou, M. 2019. Design challenges of reciprocal frame structures in architecture. *Journal of Building Enginering* 26 (100867). https://doi.org/10.1016/j.jobe.2019.100867.
(12) Popovic, O.; Chilton, J.C.; Choo, B. S. 1996. Rapid construction of modular buildings usin the reciprocal frame. *Proceedings of the International Conference on Mobile and Rapidly Assembled Structures*. Sevilla, Spain. June, 21, pp. 73 - 82.
(13) Popovic, O.; Chilton, J.C.; Choo, B.S. 1996. Sustainable roundwood reciprocal frame structures. *Proceedings of the International Conference Detail Design in Architecture*. Northampton, UK. September.
(14) Parigi, D.; Pugnale, A. 2014. Three-dimensionality in reciprocal structures: concepts and generative rules. *Nexus Network Journal* 16, pp. 151-177. https://doi.org/10.1007/s00004-014-0183-y.
(15) Popovic, O. 2014. Reciprocal frame (RF) structures: real and exploratory. *Nexus Network Journal* 16 (1), pp. 119-134. https://doi.org/10.1007/s00004-014-0181-0.
(16) Popovic, O.; Lee, D.S.H. 2014. Reciprocal frame (RF) optimized timber truss structure: a design and build case study. *WIT Transations on the Built Environment* 136, pp. 257-266. https://doi.org/10.2495/MAR140211.
(17) Garavaglia, E.; Pizzigoni, A.; Sgambi, L.; Basso N. 2013. Collapse behaviour in reciprocal frame structures. *Structural Engineering and Mechanics* 46 (4), pp. 533 - 547. https://doi.org/10.12989/sem.2013.46.4.533.
(18) Baverel, O. 2000. Nexorades: a Family of Interwoven Space Structures. Ph.D. Thesis. University of Surrey Guildford, UK. https://openresearch.surrey.ac.uk/esploro/outputs/doctoral/Nexorades-A-Family-of-Interwoven-Space/99516811502346.
(19) Popovic, O. 2008. Reciprocal frame architecture. Elsevier. Oxford, UK. ISBN: 978-0-7506-8263-3.
(20) Douthe, C.; Baverel, O.L. 2009. Design of nexorades or reciprocal frame systems with the dynamic relaxation method. *Computers & Structures* 87 (21), pp. 1296 - 1307. https://doi.org/10.1016/j.compstruc.2009.06.011.
(21) Baverel, O.; Nooshin, H.; Kuroiwa Y.; Parke G.A.R. 2000. Nexorades. *International Journal of Space Structures* 15 (2). https://doi.org/10.1260/0266351001495053.
(22) Baverel, O.; Nooshin, H.; Kuroiwa, Y. 2004. Configuration processing of nexorades using genetic algorithms. *International Association for Shell and Spatial Structures* 45 (2), pp. 99-108. https://doi.org/10.1007/s00004-007-0043-0.
(23) Baverel, O; Nooshin, H. 2007. Nexorades based on regular polyhedra. *Nexus Network Journal* 9 (2), pp. 281-298. https://doi.org/10.1007/s00004-007-0043-0.
(24) Mesnil, R., Douthe, C., Beverel, O., Gobin, T. 2018. Form finding of nexorades using the translations method. *Automation in Construction* 95, pp. 142-154, https://doi.org/10.1016/j.autcon.2018.08.010.
(25) Martínez-Arias C., Anaya-Díaz J. 2022. Parametric definition of transformable reciprocal frame structures with treebars nodes. *Informes de la Construcción* VOL 74, 565, E437, https://doi.org/10.3989/ic.86227.
(26) Houlsby, G.T. 2014. John Wallis and the Numerical Analysis of Structures. *Nexus Network Journal* 16, pp. 207-217. https://doi.org/10.1007/s00004-014-0179-7.
(27) Wallis, J. 1695. Opera mathematica. Oxford University Press. Oxford, UK.
(28) Gelez, S.; Aubry, S.; Vaudeville B. 2011. Behavior of a simple nexorade or reciprocal frame system. *International Journal of Space Structures* 26 (4). https://doi.org/10.1260/0266-3511.26.4.331.
(29) Kohlhammer, T.; Kotnik, T. 2011. Systemic Behaviour of Plane Reciprocal Frame Structures. *Structural Engineering International* 21:1, pp. 80-86, https://doi.org/10.2749/101686611X12910257102596.